Instructor Notes

So, the Test Is Positive

This is a probability-based exercise in which participants consider the implications of testing positive for a cancer when the test is less than 100% accurate.

➤ *The activity is written for workshop participants and may need modification for classroom use.*

Suggested Background Reading

- An Introduction to Probability

National Science Education Standards for Grades 5–12

Science as Inquiry

- Abilities Necessary to Do Scientific Inquiry
  Use mathematics to improve investigations and communications. Students use mathematics to perform a conditional probability exercise that considers the implication of testing positive for a cancer when the test is less than 100% accurate. Probability formulas are used during the investigation to more accurately develop explanations and communicate results.

  Formulate scientific explanations and models using logic and evidence. After formulating conclusions about the cancer testing results, students compare findings during a class discussion based on the use of scientific knowledge, logic, and evidence from the investigation.

- Understanding about Scientific Inquiry
  Mathematics is essential in scientific inquiry. In order to understand the implications of a positive cancer diagnosis, students use probability equations based on various test accuracy percentages.

Procedure Notes and Outcomes

In this exercise, participants perform the conditional probability exercise in the Activity Instructions and discuss the results.

Sample Data

Sample calculations for a test accuracy of 95% are as follows:

- the number of people in the population who actually have cancer and had a positive test

  $$\left(10{,}000 \text{ people}\right) \times \left(\frac{1 \text{ cancer}}{200 \text{ people}}\right) = 50 \text{ people in the population have cancer}$$

  $$\left(50 \text{ people with cancer}\right) \times \left(0.95\right) = \left(\begin{array}{c}47.5 \text{ people who have cancer}\\ \text{and test positive for the disease}\end{array}\right)$$

- the number of people in the population who do not have cancer but had a positive test

  $$\left(\begin{array}{c}10{,}000 \text{ people}\\ \text{in the population}\end{array}\right) - 50 \text{ who have cancer} = \left(\begin{array}{c}9{,}950 \text{ people in the population}\\ \text{who don't have cancer}\end{array}\right)$$

  $$\left(9{,}950 \text{ people}\right) \times \left(0.05\right) = 497.5 \text{ people who test positive for cancer but don't have it}$$

- the total number of positive tests

  $$47.5 + 497.5 = 545 \text{ total positive tests in the population}$$

- the conditional probability of having cancer (given you tested positive) using the previously calculated values and the following equation:

  $$\left(\frac{\text{number of people with cancer and a positive test}}{\text{total number of positive tests}}\right) \times 100$$

  $$\left(\frac{47.5}{545}\right) \times 100 = 8.71\%$$

Plausible Answers to Questions

1. What are the implications of these calculations to your diagnosis? Discuss the results as a class.

   Answers will vary. Based on their calculations, participants should be cautiously optimistic.

2. How does the picture change if the test given were 98% accurate? or 99%? or 99.9%?

Conditional Probability Data				
accuracy	number of people with cancer who test positive	number of false positives	total number of positive tests	conditional probability
98%	49.00	199.00	248.0	19.75%
99%	49.50	99.50	149.0	33.22%
99.9%	49.95	9.95	59.9	83.33%

Extension

Consider related issues such as the implications of testing negative and actually having the disease, or how chances of actually having the disease change if a second test with the same probability comes out positive.

Reference

Paulos, J.A. *Innumeracy;* Vintage: New York, 1988; pp 89–90.

Activity Instructions

So, the Test Is Positive

What does it really mean to have a positive result to a medical test?

Class Discussion

Consider the following scenario:

Published research documents the fact that 1 out of 200 people in a population in a certain region of the country have a certain type of cancer. A sample of 10,000 people from this area have taken a specific test for this cancer. This test is 95% accurate—this means that if someone has cancer, the test will be positive 95% of the time, and if someone does not have cancer, the test will be negative 95% of the time.

Assume you were diagnosed with cancer using this test. What would this mean? Calculate the following and then reconsider your answer to this question:

- the number of people from the population who actually have cancer and had a positive test
- the number of people in the population who do not have cancer but had a positive test (These tests are called false positive.)
- the total number of positive tests
- the conditional probability of having cancer (given you tested positive) using the previously calculated values and the following equation:

$$\left(\frac{\textit{true positives}}{\textit{total number of positives}}\right) \times 100$$

Questions

1. What are the implications of these calculations to your diagnosis? Discuss the results as a class.
2. How does the picture change if the test given were 98% accurate? or 99%? or 99.9%?